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Accelerated expansion of the universe and scalar tensor theory of gravity

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Abstract : The possible role of a scalar field, non minimally coupled to gravity, in the search for dark energy is very briefly reviewed Some possibilities in the framework of Brans-Dicke theory are discussed

Keywords : Brans-Dicke theory, scalar field

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1. Introduction

Observations now have firmly established that our universe is expanding at an accelerated rate The initial indications came from luminosity-redshift surveys of some type-Ia super-novae [1], which showed that they appear to be dimmer than they should have, requiring a negative value of the dimensionless deceleration parameter q. This parameter is defined as $q = -(a\ddot{a}/\dot{a}^2)$, a being the scale factor of the universe, a measure of the linear dimension of the universe This result has later been firmly supported by other high precession data [2].

This defies all intuition as the dynamics of the universe is governed by gravity and gravity is attractive The natural outcome of this observation is indeed an urgent search for some field which can give rise to an accelerated expansion Certainly a host of different kinds of fields appear and claim their suitability as a driver of this acceleration. As most of the matter present in the universe with the normal attractive gravitational property is invisible and hence called the dark matter, this new brand of matter with a wrong sign of gravity is generically called the Dark Energy. Amongst the dark energy candidates the first and perhaps the most talked about one is the rejuvenated cosmological constant Λ. The other examples are a scalar field with a positive potential (this form is generically called the quintessence field), a Chaplygin gas, non linear contribution of the Ricci curvature scalar in Einstein's equations, and many others. There are excellent reviews, for example, see [3] In most of the cases the idea is to take advantage of the fact that unlike in Newtonian gravity, in general relativity pressure also plays a similar role as matter density in the determination of the gravitational intensity. As our universe is homogeneous and isotropic and hence is correctly described by a Robertson-Walker metric, Einstein equations look like

$$3\left(\frac{\dot{a}^2+k}{a^2}\right) = 8\pi G\rho \tag{1}$$

and

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2+k}{a^2} = -8\pi Gp, \tag{2}$$

where ρ and p are the density and pressure of the matter present, G is the Newtonian constant of gravity and k is the curvature index having possible values amongst 0 and ±1. An overhead dot represents differentiation with respect to the cosmic time t. These two equations will yield

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p). \tag{3}$$

Obviously, if the pressure is sufficiently negative, so that $\rho + 3p < 0$, one can have a negative value of q. The dark energy models are such that they produce an effective negative pressure, which drives the accelerated expansion, *i.e.*, gives rise to a negative value of q

Observations further tell us that the accelerated expansion is rather a recent phenomenon, having started at quite a matured age of the universe when it looked very much like today, *i.e.*, dominated by matter like galaxies and dark matter [4] This observation indeed came as a relief as only a decelerated expansion provides the perfect ambience for the synthesis of matter in the form of atomic nuclei in the radiation dominated era as also for the formation of galaxies in later stages

The dark energy candidates all serve the primary purpose, *i.e.*, they produce an accelerated expansion during the matter dominated era, some of them can indeed produce the signature flip of q in a recent past but perhaps none of them has a sound theoretical basis regarding why it should be there

In the absence of a final verdict in favour of any of the dark energy models, scalar tensor theories also stake their claims as a possible arena where the answers to this challenge can be sought Amongst scalar tensor theories, Brans-Dicke theory [5] has been successful to have an interest alive for some reasons One important appeal of this theory had been that it believed to have general relativity as a limit of this theory for a very high value of a dimensionless parameter ω This has now been shown to be true only in a restricted sense [6], but the theory proved itself to be useful in finding solutions to many a cosmological problem For example, when the inflationary models were all struggling to find a solution to the graceful exit problem, Brans-Dicke theory came up with the idea of an extended inflation [7] The dark energy problem also has some solutions in Brans-Dicke theory or in some modification of the same In this article, we shall briefly describe some of the attempts towards this This is by no means exhaustive Only some work will be described where the present author had been involved in The next section describes some work in the theory proper and in the third section investigations in some modification of the theory will be talked about

2. Acceleration of the universe in Brans-Dicke theory

The field equations in Brans-Dicke theory along with a quintessence scalar field ψ will be

$$3\frac{\dot a^2+k}{a^2}=\frac{1}{\phi}\rho_m+\rho_\psi+\frac{1}{2}\omega\frac{\dot\phi^2}{\phi^2}-3\frac{\dot a\dot\phi}{a\phi}, \tag{4}$$

$$2\frac{\ddot a}{a}+\frac{\dot a^2+k}{a^2}=\frac{p_\psi}{\phi}-\frac{1}{2}\omega\frac{\dot\varphi^2}{\phi^2}-2\frac{\dot a\dot\varphi}{a\phi}-\frac{\ddot\varphi}{\phi}, \tag{5}$$

where ρ_m is the density of normal matter and ρ_ψ and p_ψ are the density and pressure due to the quintessence scalar field ψ and they are given by

$$\rho_\psi=\frac{1}{2}\dot\psi^2+V(\psi) \tag{6}$$

and

$$p_\psi=\frac{1}{2}\dot\psi^2-V(\psi). \tag{7}$$

The equations are written in units where $8\pi G = 1$ $V(\phi)$ is the potential of the quintessence field Both the scalar fields ϕ and ψ will satisfy the respective wave equations in view of which the field equations will yield the conservation of matter as

$$\rho_m=\frac{\rho_0}{a^3},$$

ρ_0 being a constant The pressure of the normal matter is taken to be zero so as to be consistent with the present matter dominated universe This set of equations indeed gives various possibilities [8]. For different choices of the potential $V(\psi)$, the model can yield power law solutions for the scale factor a as

$a = a_0t^n$

where $n > 1$ so that $q = -(n-1)/n$ has a negative value For example, for a spatially flat universe ($k = 0$), $V(\psi) = V_0\psi^4$ yields $n = 4/3$, *i.e.*, $q = -1/4$. For this rate of acceleration the model works for the entire time span, *i.e.*, $0 < t < \infty$ For a faster rate of acceleration, however, the model works only upto a finite future, $0 < t < t_{max}$ The model can produce non-decelerating solution with nonzero spatial curvature ($k \neq 0$) as well, but those are consistent only with $q = 0$, *i.e.*, a coasting universe

As a bye-product of the model, it is also shown that with some choices of the parameters, $k = 0$ is an attractor, *i.e.*, the flatness problem has some solution in this model

The most attractive feature of working in Brans-Dicke theory is that it can produce an accelerated expansion for the spatially flat universe even in the absence of any quintessence field [9]. If we put ρ_ψ and p_ψ equal to zero in the field equations, a power law solution for the scale factor can be obtained as $a = a_0t^n$ where $n=\frac{2(\omega+1)}{3\omega+4}$ For $\omega = -5/3$, the deceleration parameter $q = -1/4$ Consistency of the equations demand that $-2 < \omega < -3/2$

and this range of values for ω yields different rates of acceleration This result is quite interesting as the desired acceleration can be driven by the Brans-Dicke scalar field, which is already there in the purview of the theory and one does not have to invoke any quintessence field by hand

But the problem in these models are two-fold They show only an acceleration, the relevant values of ω fail to show a signature flip in q in the matter dominated era The second problem is that these accelerated models all need a very low negative value of ω, of the order of unity Whereas local astronomical observations clearly indicate that this value should be very high [10]

3 Acceleration in modified Brans-Dicke theories

Brans-Dicke scalar field is non minimally coupled to gravity and has an interference term with the Ricci scalar R in the action as follows

$$A = \int\left(\phi R + L_m\sqrt{-g}\right)d^4x, \tag{8}$$

where L_m is the lagrangian for any field other than the Brans-Dicke field It is amply clear that ϕ does not have any direct interaction with matter of any form Now if the theory is modified so that ϕ has an interaction with matter as well, one arrives at many possibilities Two examples will be described in this section in case of a spatially flat universe (k = 0)

If it is assumed that the universe is filled with a pressureless fluid consistent with the present universe but the matter does not conserve itself and rather has an interaction with the Brans-Dicke field, then the conservation equation of matter becomes

$$\dot{\rho} + 3H\rho = Q, \tag{9}$$

where Q takes care of the interaction between matter and the scalar field and actually is a measure of the rate at which energy is pumped in from one form to the other Q = 0 obviously indicates an absence of interaction The field equations in this case is rather involved But with a conformal transformation $\bar{g}_{\mu\nu} = \phi g_{\mu\nu}$, the equation system looks more tractable like

$$3\frac{\dot{\bar{a}}^2}{\bar{a}^2} = \bar{\rho} + \frac{2\omega+3}{4}\dot{\lambda}^2, \tag{10}$$

$$2\frac{\ddot{\bar{a}}}{\bar{a}} + \frac{\dot{\bar{a}}^2}{\bar{a}^2} = -\frac{2\omega+3}{4}\dot{\lambda}^2, \tag{11}$$

and the matter conservation equation becomes

$$\dot{\bar{\rho}} + 3H\bar{\rho} = \bar{Q} \tag{12}$$

An overhead bar obviously denotes quantities in the transformed version and $\lambda = \ln\phi$ It can be shown that if we take ω to be a constant, then one can have accelerated expansions, but there is no sign flip in q like the models in the previous section But if the theory is generalized to incorporate a varying ω [11], there is obviously the possibility that the universe enters the accelerated phase only in the recent past [12] The scale factor a is not a simple power function of time, and q is given by

$$q = -1 + \frac{\frac{2}{3+\alpha}(1-\phi_0 t)^2 - \phi_0^2 t^2}{\left[\frac{2}{3+\alpha}(1-\phi_0 t) - \phi_0 t\right]^2} \tag{13}$$

where α is small dimensionless constant The deceleration parameter as given by this equation indeed has a signature flip in the recent past The era at which q enters its negative phase is seen to be not crucially sensitive to the value of α

The problem of this model is that the interaction between the scalar filed and matter is not well understood But this indeed opens up a possibility, as this model does not require a stringent restriction on the value of ω, and even high values of ω can be quite consistent with an accelerated expansion of the universe

Another interesting possibility is to use a quintessence scalar field in Brans-Dicke theory as described in the previous section but with an interaction between ϕ and the quintessence field ψ which slowly roles down a potential $V = V(\psi)$ The interaction is such that the effective potential looks like

$$U(\psi,\phi) = \phi^{\beta}V(\psi), \tag{14}$$

so that the Brans-Dicke field, which is already there in the theory, helps universe enter the accelerated phase quite late in the history of the evolution. Here, β is a constant. This potential U replaces $V(\psi)$ in eqs (4) and (5) The motivation comes from the following example It is quite well known that an inflationary expansion, *i.e.*, an accelerated expansion of the universe was required in the early phase of the evolution so as to solve certain problems like that of flatness (or fine tuning) and horizon problem amongst others The problem of these inflationary models was that of a graceful exit, how the universe can get out of this inflationary expansion to enter a more sedate decelerated expansion. The idea of an extended inflation came out in investigations in Brans-Dicke theory where a scalar field, slowly rolling down its potential, could indicate the possible solution of this graceful exit problem [7] It was also shown that a potential as in eq (14) could indeed produce an inflation in the early stage and the inflaton field had an oscillation during the later phases marking an end to the inflationary stage [13] The recent problem is just the reverse of that, a graceful entry so to say, where the quintessence field should have an oscillatory behaviour in the beginning and grows during the later stages to mark the onset of an accelerated expansion

It is shown that indeed the Brans-Dicke theory can give rise to this kind of a situation With some conditions on the constants of the model and some arbitrary constants of integration, the quintessence field ψ can have an oscillatory behaviour to start with and grows later to drive the accelerated phase [14] For an exponentially expanding universe, the potential V should be an exponential function of ψ However, for a power law expansion with $n > 1$ as in the previous section, the form of the potential V could be quite arbitrary This is quite an interesting feature, as the form of the potentials used to drive the accelerated expansion of the universe, as mentioned earlier, are not based on a sound background physics, here this feature is generic, quite independent of the choice of the potential Also, the value of ω is not necessarily restricted to a very low value, so the theory can work for the local astronomical results as well.

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